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Evaluation of Real Options with Information Costs

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ABSTRACT

This paper presents a simple framework for the analysis, valuation and simulation of several real options in the presence of shadow costs of incomplete information. Information costs can be viewed as sunk costs in the spirit of Merton's (1987) model of capital market equilibrium with incomplete information. We incorporate these sunk costs in standard discounted cash flow techniques and present the basic concepts of real options. The justification of information costs in real projects is based on the observation that R&D needs to be done before investment decisions. These costs account for all the expenses needed to be informed about an investment opportunity and the management of projects. This analysis extends the models in Bellalah (1999, 2001) for the valuation of real options within information uncertainty. We present valuation procedures and simulations for the values of common real options in the presence of shadow costs of incomplete information.

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I. INTRODUCTION

A company's value creation is determined by resource allocation and the proper evaluation of investment alternatives. Managers make capital investments to create future growth for shareholders. Investments lead to patents or technologies, which open up new growth possibilities. In general, managers use the basic investment techniques as the capital asset pricing model (CAPM), the cost of capital and the discount cash flow techniques, DCF. In investment valuation, organisations also use quantitative approaches such as net present value (NPV), decision tree analysis (DTA), payback time, or scenario/simulation which do not account for intangible factors such as future competitive advantage, future opportunities, managerial flexibility, the strategic value of the investment, etc. This is because the expected outcomes are not easy to forecast and the variability of investment returns may be extremely high. New techniques for capital budgeting incorporate real options, active management, and strategic interactions between investment and financing decisions.[1]

Information plays a central role in the capital budgeting process and in investment and financing decisions. Edwards and Wagner (1999) study the role of information in capturing the research advantage and how to incorporate information into the decision process of active investment management. They show that implementation costs make sense only when weighed against the benefit of enhanced performance. They recognise that the most valuable commodity in the market is information that reduces uncertainty. In this spirit, trading cost information is part of the research that gives a manager active advantage. Edwards and Wagner (1999) show that managers must measure and develop confidence in the value of their research and then incorporate feedback from the market.

Merton (1987) adopts most of the assumptions of the original CAPM and relaxes the assumption of equal information across investors. He assumes that investors only hold securities of which they are aware. In his model, the expected returns increase with systematic risk, firm-specific risk, and relative market value. The expected returns decrease, however, with relative size of the firm's investor base, referred to in Merton's model as the "degree of investor recognition". The intuition behind Merton's model is that investors consider only a part of the opportunity set, that full diversification is not possible, and that firm specific risk is priced in equilibrium. The main distinction between Merton's model and the standard CAPM is that investors invest only in the securities about which they are "aware". This assumption is referred to as incomplete information. The more general implication is that securities markets are segmented. The intuition behind this result is that the absence of a firm-specific risk component in the CAPM comes about because such risk can be eliminated (through diversification) and is not priced. It appears from Merton's model that the effect of incomplete information on expected returns is greater than the highest specific risk of the firm and the highest weight of the asset in the investor's portfolio. The effect of Merton's non-market risk factors on expected returns depend on whether the asset is widely held or not.[2]

Kadlec and McConnell (1994) document the effect of share value on the NYSE and report the results of a joint test of Merton's (1987) investor recognition factor and Amihud and Mendelson's (1986) liquidity factor as explanations of the listing effect.

The cross-sectional regressions provide support for investor recognition as a source of value from exchange listing. The regressions support Merton's model. The results also provide support for superior liquidity as a source of value from exchange listing. They provide support to Amihud and Mendelson (1986) model.

Foerster and Karolyi (1999) construct an empirical proxy for the shadow cost of incomplete information for each firm, using the methodology in Kadlec and McConnell (1994). The investor recognition hypothesis of Merton suggests that abnormal returns may be due to the changes in the shareholder base, adjusted by the stock's residual variance and relative size. The results obtained by Foerster and Karolyi (1999) are supportive of the Merton (1987) hypothesis and consistent with Kadlec and McConnell (1994).

Coval and Moskowitz (1999) document the economic significance of geography and attempt to uncover the effect of distance on portfolio choice. They find that local equity preference is strongly related to firm size, leverage and output tradability. Their results suggest an information-based explanation for local equity. This is consistent with the findings in Kang and Stulz (1997) who find that foreign investors underweigh small, highly levered firms, and firms that do not have significant exports. These results may be a response to severe information asymmetries associated with these firms.

Brennan and Cao (1997) develop a model of international equity portfolio investment flows which is based on the differences in informational endowments between foreign and domestic investors. The authors show that when domestic investors possess a cumulative information advantage over foreign investors about their domestic market, investors tend to purchase (sell) foreign assets in periods when the return on foreign assets is high (low).

Stulz (1999) examines the effect of globalisation on the cost of equity capital and argues that this cost decreases because of globalisation. The empirical evidence gives support to the theoretical prediction that globalisation decreases the cost of capital. He gives strong theoretical arguments justifying why the cost of capital should fall when markets become more open to foreign investors. Following Merton (1987), Stulz (1999) assumes that some investors do not hold some securities because they do not know about them. He provides a model in which this assumption amounts to attributing the home bias to ignorance or a non-modelled behavioural bias. This leads Stulz (1999) to show that the impact of globalisation on the cost of capital depends heavily on the extent of the home bias. However, the empirical evidence in Stulz (1999) shows that the effect of globalisation on the cost of capital is rather small because of the home bias.

Merton's (1987) model shows that asset returns are an increasing function of their beta risk, residual risk, and a decreasing function of the available information for these assets. Amihud and Mendelson (1988) consider several observed corporate policies that can be viewed as increasing the liquidity of investments. Their suggested policies include going public, instituting limited liabilities on equity claims, listing on organised exchanges, distributing ownership among many shareholders, etc. Since the transmission of this information is costly as in Merton's model, Amihud and Mendelson (1988) show how managers can balance the costs against the added value from the higher liquidity of the claims of the firm.

The above literature reveals the importance of information costs in the pricing of financial and real assets. Using this framework, Bellalah and Jacquillat (1995) and Bellalah (1999) develop simple models for the pricing of financial options in the presence of information costs. A similar analysis can be extended to real options using the same methods as in Bellalah (2001). Our work extends the standard capital budgeting techniques by accounting for the dynamic dimension of existing theories. The main objective is to analyse numerically the real option approach in capital budgeting investment decisions and compare this approach to the traditional *NPV*. This limits the study to only one stochastic underlying variable: the cash inflows.[3]

This paper is organised as follows. Section 1 reminds the use of traditional capital budgeting models. It incorporates also information costs in standard discounted cash flow techniques. Section 2 analyses the basic concepts and specific features of real options. Section 3 deals with valuation and simulation of real options. It suggests an extension and an adaptation of Black-Scholes (1973) model, Merton (1998) model and the binomial approach for the valuation of real options by accounting for the effects of incomplete information. Two cases are analysed: the case when the underlying asset is observable and the case when it is neither observable nor continuously traded. Simulation results are proposed to show the impact of information costs on real options values.[4]

II. TRADITIONAL MODELS AND REAL OPTIONS

A. Traditional Capital Budgeting Models and Information Costs

Investment decisions are often made with reference to standard discounted cash flow techniques, (DCF analysis). The most common capital budgeting models used by corporations involve either the basic net present value (NPV), Scenario/Simulation, or Decision Tree Analysis (DTA).

The NPV is the sum of the expected future cash flows minus the initial costs of investments. This method seems to give better results than the accounting rate of return (ARR), the profitability index (PI), the internal rate of return (IRR), the modified internal rate of return (MIRR), and the payback method. However, this method ignores flexibility, assumes that the investment either falls into a reversible or an irreversible category, and that managers are given unbiased expected cash flows. For ease of exposition, the following notations are used: $E_P(CF_t)$: expected cash flow; R: risk adjusted discount rate; r: risk-free discount rate; $\overline{CF}_t$: certainty equivalent cash flow; I_0 : investment outlay at time 0; T : time to maturity; and λ_s , (λ_c): information cost regarding the firm's cash flows (and the real option).

In the presence of information costs, the *NPV* can be written as:

$$NPV = \sum_{t=1}^{T} \frac{E_P(CF_t)}{(1+R+\lambda_s)^t} - I_0 = \sum_{t=1}^{T} \frac{\overline{CF}_t}{(1+r+\lambda_s)^t} - I_0$$

It is important to note that the information cost appears as an additional discount rate in the discounting of risky streams. This is the main intuition in Merton's (1987) model. In fact, this cost reflects the additional return required by investors to get compensated for their investments in information. An investor does not invest in a real project if he does not know about that project. The process of information acquisition has a cost that must be accounted for in the computation of the present value of cash flows. If the manager pays 2 million in the process of information acquisition and the investment is equal to 100 million, then he must require at least 2/100 or 2% as an additional return above the riskless interest rate. Hence, instead of a discount rate r, a new discount rate equal to (r + 2 %) must be used as a rough approximation in this case.

Several managers rely on sensitivity analysis using high, low, or medium scenarios to bind the uncertainty. This method tends to show the impact on NPV and its sensitivity to each variable. Then the resulting NPV values are recorded. It assumes that other variables are constant in scenario base of their expected values. This technique recognises the existence of uncertainty but does not capture the flexibility due to "uncertainty" and offers little managerial guidance in investment decision process. In this analysis, information costs can be easily introduced in the simulation of the present values of risky streams in the same way as we have done for the calculation of the NPV.

The Monte Carlo simulation is not biased when modelling cash flows and deciding on the values for the relevant variables and correlation. For each variable, a probability distribution is designated and the cash flows are simulated discretely. Then, they are used to calculate the NPV. However, the serial dependency is complex to quantify. The NPV distribution given by the simulation is also hard to interpret economically (Trigeorgis, 1990 and 1993). This method is useful in the calculation of projects under uncertainty, even though, it has its proper limits. Information costs can also be easily integrated in this analysis in the discounting of the risky streams.

The Decision Tree Analysis approach takes into account later decisions and incorporates some of the manager's flexibility into the valuation process. Investments are divided into a series of sub-investments that will be undertaken at different stages. The implementation of these investments in the future will depend upon some future event, thus enabling managers to decide whether to invest further or not. This process can not be implemented without additional information. This leads necessarily to information costs in the spirit of Merton (1987).

B. Analysis of Real Options

During the last decade real options have been given increasing interest by corporate practitioners in industries where the projects are costly and uncertain. Companies allocate resources for existing businesses or new ventures, and managers decide whether to invest now, to do nothing or to wait. When valuing investment decisions, the options to abandon or to defer, the options to expand or to switch are embedded into the project. These implicit options occur naturally or may be planned at some flexibility.[5]

Investment decision-making seems to be justified as a way to account for flexibility and can be thought of in terms of real options (Dixit and Pindyck, 1994).

Option pricing theory evaluates the firm as its operating options were managed optimally, without future information on optimal choices to be made. A distinction must be made between real assets, (which have a market value) and real options, (which consider the opportunities to purchase future real assets on favourable terms).[6]

Investment is defined in financial economics as the act of incurring an immediate cost in the expectation of future rewards (Dixit and Pindyck, 1995). The initial outlay is a payment for a right with no obligation to undertake a project. Real options give the right to receive a future cash flow from the investment cost. This is equivalent to a standard call option on a real asset. Using the option theory, the company can be viewed as a future possibility where an investor pays a premium for the right to buy a specific stock at a known exercise price at a certain time in the future. The investment amount is then the strike price, allowing the investor to capture the value of the underlying project (Trigeorgis, 1990 and 1993). A real option strategy forces managers to compare every opportunity arising from existing investments with the full range of opportunities open to them. It promotes strategic leverage and encourages managers to exploit situations where investment can keep their company in the game. The strategy reduces the upside as well as the downside risk, and empowers managers to defer the investment opportunity without increasing the exercise price.

Real options can be used by managers with a basic understanding of option pricing models and tools. As they are important in strategic and financial analysis, they can be a complement to the standard NPV valuation. The NPV ignores the value of flexibility and creates a static picture of existing investments and opportunities. The traditional techniques treat opportunities as a "now or never" investment even if many investments can be deferred in the future without loosing their value.

There is a large scope for applications of option pricing techniques for valuation of an entire firm.[7] A real option confers flexibilities to its holder as the option to invest, to wait, to divest, etc.[8] These options can be economically important. The decision about when to invest is analogous to the decision about when to exercise an American call. The sensitivity of the value of the firm to these possibilities makes a real option valuation method better than the standard NPV. This is because an ordinary NPV valuation predicts future cash flows according to today's information. By using the real option's approach, the value of a company corresponds to the value of a portfolio of operating options yielding a stream of future cash flows. This portfolio can be seen as a portfolio of financial options on those future cash flows.

There are totally irreversible investments (where the whole investment cost is lost at the end of the operating phase), and partially irreversible investments (whose value can be partially recovered). Irreversibility can also arise from government regulations which make investments irreversible. An irreversible investment opportunity is like a standard call option even if the asset can be sold to another investor. Two types of uncertainty are present in capital investments: economic uncertainty and technical uncertainty each with a positive increase effect on the value of a real option. The economic uncertainty is correlated with the actual exogenous movement of the economy: interest rate, inflation, industry prices, etc. This uncertainty could be reduced by waiting for new information before making the final investment. The technical uncertainty is the uncertainty in the project itself. It is endogenous to the decision process and is affected by management. For example, the uncertainty in the outcome of a R&D project can only be reduced with an actual step by step investment, until the future technical uncertainty is resolved (Dixit and Pindyck, 1994).

The analogy between financial and real options also has its limitations. There are three factors that make a real option different from a financial option: the proprietary state, the complex characteristics, and non tradability of real options (Kester, 1993). Firstly, all financial options are proprietary, and the holder decides when the option should be exercised. Real options would present a proprietary characteristic when the company has a unique and exclusive know how in a technological process or has access to a patent. In general, investment opportunities with barriers to entry serve as proprietary real options. This is not the case when investment opportunities are shared by competitors and other participants. Secondly, most financial options are derived from the underlying asset. Some real options have more complex characteristics. They give the holder the right not only to receive the gross present value of the future cash flows from the investment, but also investment opportunities in the future. In this case, the option becomes compounded and written on many another options. Thirdly, when compared to the financial options markets, the real options markets are imperfect and only some proprietary real options can be traded with high transaction costs and few participants (Trigeorgis, 1990 and 1993). Shared real options cannot be tradable on the market since they are already a public good for the whole industry.

III. VALUATION AND SIMULATION OF REAL OPTIONS IN THE PRESENCE OF INCOMPLETE INFORMATION

A. Valuation Procedure in the Presence of Information Costs in a Continuous-time Setting

The valuation of financial options is based on the fact that an option can be replicated by a portfolio of traded securities. Since this equivalence is not dependent on risk attitudes, the value of the expected future payoffs can be derived from a risk-neutral approach and discounted at the risk-free interest rate. This concept can also be applied to real options, even if they are not traded in financial markets. The fundamental assumption is that a non traded project has the value that it would have had if it were traded in the financial markets (Smith and Nau, 1994).

Trigeorgis (1990, 1993) shows that in the DCF analysis the discount rate is received by identifying a twin security for each project. The twin security has the same risk characteristics as the specific project and is traded in financial markets. In this context, the option analogy could use the same twin security to replicate a non-arbitrage portfolio. Given the price of the project's twin security, management can, in principle, replicate the returns to a real option by purchasing a certain number of shares while financing the purchase partly by borrowing at the risk-free rate. This makes the application of risk neutral valuation techniques for traded and non traded assets possible. The derivation of the standard formulas for option pricing in the presence of information costs appears in Bellalah (1999, 2001).

1. A General Derivation of the Values of Real Options

The use of option valuation techniques in the valuation of real assets is based on some important assumptions.[9] In general, individual values of real options are non-additive and the combined value could be complex to compute. Kulatilaka (1993) shows that the combined value of interacting options could either be higher or lower than the sum of the individual values. The combined value is dependent on the type of options, the degree of separation, the degree of being "in the money", and the order of the options involved. Trigeorgis (1990, 1993) describes the interaction between options as basically additive. This is the case when the interacting options are of different types, i.e. calls and puts. He gives an example on the interaction between the option to abandon (which is equivalent to a put) and the growth option (which is equivalent to a call). He shows that these two options are additive because they are of different types.

a. Real option inputs

Because the value of a real option is determined by seven parameters, exploiting proactive flexibility is a question of pulling one or more parameters. To extend the Black & Scholes (1973) model and the binomial model to a context taking into account the presence of shadow costs of incomplete information, seven input parameters are required: expected cash inflows and cash outflows, the annual cost (or value lost over the duration of the option), the risk-free interest rate, the level of uncertainty, changes in the duration, and information costs.

Gross present value of the project, V, is the value of the expected cash flows to be received from the investment. It is considered significant without the investments. A higher present value of expected operating cash inflows can be achieved by increasing revenues, raising the price earned, producing more, or by generating compound business opportunities. The economic uncertainty is assumed to influence the gross present value and thus make it follow a geometric Brownian motion with a random part determined by the standard Wiener process dz(t).

The capital investments to be made, I, is the present value of the fixed costs over the lifetime of the investment. It is equivalent to the exercise price of a financial option. Here, we suppose certain capital investments. The reduction of the expected operating cash outflows can be achieved by leveraging economies of scale or by leveraging economies of scope in partnership.

The dividends,[10] **δ,** are sums paid regularly to stockholders. This could be the costs incurred to preserve the option by keeping the opportunity alive, or the cash flows lost to competitors that go ahead and invest in another opportunity. The cost of waiting could be high if an early entrant were to seize the initiative. The dividends are correspondingly high, thus reducing the option value of waiting and the value lost to competitors can be reduced by discouraging them from exercising their options. This is the case for example in locking up key customers or lobbying for regulatory.

The risk-free interest rate, r, corresponds to the interest rate for a risk-free bond with the same expiration date as the project. Expected increase in the interest rate raises the option value, despite its negative effect on NPV (reduces the PV of the exercise price).[11]

The volatility, σ **,** is the standard deviation of the growth rate of the value of future cash inflows. This is the crucial difference from NPV analysis. When uncertainty of expected cash flows rises it increases the value of flexibility. For a project it could be a little more complex to find the correct volatility when compared to financial options.

Time to maturity, T, corresponds to the time left until the opportunity disappears. It depends on technology (products life cycle), competitive advantages (intensity of competition), and contracts (patents, leases, licences). The time to maturity, is subjectively defined by management as the time it takes for competitors to exploit the same opportunity.[12] An increase in the opportunity's time raises the option's value because it increases the total uncertainty. The company might be able to extend its option by extending exclusive raw material supply contracts, locking up distribution channels, etc.

The information costs, λ **,** are the costs engaged by investors to get informed about the projects and their real options. We make a distinction between information costs related to the underlying project cash flows, λ_V, and information costs related to each implicit real option, λ_C.

b. Valuation of real options when the underlying asset is observable under incomplete information

Consider the following dynamics of the project's value:

$$\frac{dV}{V} = \mu dt + \sigma dz \tag{1}$$

where μ and σ refer to the instantaneous rate of return and the standard deviation of the project, and dz is a geometric Brownian motion. Let X be the price of a dynamic portfolio of assets perfectly correlated with V:

$$\frac{dX}{X} = \alpha dt + \sigma dz \tag{2}$$

where α stands for the expected return from owning a completed project. Let $\delta = \alpha - \mu$. In this context, δ represents an opportunity cost of delaying investment. If δ is zero, then there is no opportunity cost to keeping the option alive. Hence, the value of δ must be positive. Let G(V) be the value of the firm's option to invest. Using Merton's (1987) model, Bellalah and Jacquillat (1995) and Bellalah (1999, 2001) obtain option prices in the context of incomplete information.

Consider a portfolio: long an option which is worth G(V) and go short G_V units of the project. The value of this portfolio is:

$$P = G - G_V V \tag{3}$$

Since the short position includes G_V units of the project, it requires the paying out of an amount $\delta\, G_V V$. The total return for this portfolio over a short interval of time, dt, is:

$$dP = dG - G_V\, dV - \delta\, G_V\, V\, dt \tag{4}$$

Since there are information costs embedded in the option and its underlying assets, the return must be equal to $(r + \lambda_V)$ for the project and $(r + \lambda_C)$ for the option, where λ_V and λ_C refer respectively to the information costs on the project and the option. In this context:

$$dP = (r + \lambda_C)\, G\, dt - (r + \lambda_V)\, G_V\, V\, dt \tag{5}$$

Assuming that a hedged position is constructed and since the application of Itô's lemma, the value of dG is:

$$dG = \frac{1}{2}\sigma^2 V^2 G_{VV} dt + \mu V\, G_V\, dt + \sigma V\; G_V dz + G_t dt \tag{6}$$

Substituting dV and dG, given respectively by relations (1) and (6), in equation (4), we get after simplification:

$$dP = (\frac{1}{2}\sigma^2 V^2 G_{VV} - \delta\, V\, G_V + G_t)dt \tag{7}$$

When the time to maturity of the option is finite, this equation becomes:

$$\frac{1}{2} G_{VV}\, \sigma^2 V^2 + (r + \lambda_V - \delta)\, V\, G_V - (r + \lambda_C)\, G + G_t = 0 \tag{8}$$

For the valuation of standard calls, under the following condition:

$$G = \max\,(V - I, 0) \tag{9}$$

The call value is given by:

$$G = V\, e^{-(\delta+\lambda_C-\lambda_V)T} N(d_1) - I\, e^{-(r+\lambda_C)T} N(d_2) \tag{10}$$

with:

$$d_1 = \frac{\left[\ln\left(\frac{V}{I}\right) + \left(r + \lambda_V - \delta + \frac{1}{2}\sigma^2\right)T\right]}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}\,.$$

This formula constitutes an adaptation of the Black and Scholes (1973) model to valuing the real options in the context of incomplete information.[13]

c. Valuation of real options when the underlying asset is neither observable nor continuously traded under incomplete information

Using the same analysis as in Merton (1998) and following the same approach as above, the equivalent of equation (28) in Merton (1998) is:

$$\frac{1}{2}\nu^2 V^2 G_{VV} + (r + \lambda_V - \delta)V\,G_V - (r + \lambda_C)\,G + G_t = 0 \quad (11)$$

where ν^2 is the variance of the V-Fund portfolio in Merton (1998).

This equation can be solved under the following condition:

$$G(V,T) = E[h(VY)]$$

where Y is a log-normally distributed random variable with E(Y) = 1 and variance of ln(Y) is equal to $\theta^2 T$ and E(.) is the expectation operator over the distribution of Y.

The solution to this equation when h(V) = max(V – I, 0) is given by:

$$G = V\exp((\lambda_V - \lambda_C)\,T)\,N(d_{11}) - I\exp(-(r + \lambda_C)\,T)\,N(d_{11} - \sqrt{\gamma}) \quad (12)$$

with:

$$d_{11} = \frac{\left[\ln\left(\frac{V}{I}\right) + (r + \lambda_V)T + \frac{\gamma}{2}\right]}{\sqrt{\gamma}}, \qquad \gamma = \nu^2 T + \theta^2 T.$$

When compared to formula (10), this formula allows understanding the effect of the underlying asset price to not be observable. The main difference in the option pricing formula with and without continuous observation of the underlying asset is that the variance of the underlying does not go to zero around the maturity date because of the "jump" event at expiration. This formula can be applied when the underlying asset is neither continuously traded nor continuously observable. This is a simple generalization of formula (27) in Merton (1998) to account for the effects of incomplete information.

2. The Value of the Option to Invest

The value of the option to invest under incomplete information can be computed using the following equation:

$$\frac{1}{2}\sigma^2 V^2 G_{VV} + (r + \lambda_V - \delta)\,V\,G_V - (r + \lambda_C)\,G = 0 \quad (13)$$

This equation for the value of G(V) must satisfy the following conditions:

$$G(0) = 0, \qquad G(V^*) = V^* - I, \qquad G_V(V^*) = 1$$

The value V* is the price at which it is optimal to invest. At that time, the firm receives the difference V* - I. Following Bellalah (2001), the solution to the differential equation is:

$$G(V) = aV^{\beta} \tag{14}$$

where:

$$\beta = \frac{1}{2} - \frac{r + \lambda_V - \delta}{\sigma^2} + \left[\left(\frac{r + \lambda_V - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r + \lambda_C)}{\sigma^2}\right]^{0.5}$$

and:

$$a = \frac{V^* - I}{V^{*\beta}}, \qquad V^* = \frac{\beta I}{\beta - 1}$$

Table 1 simulates the value of the investment opportunity G(V), given by equation (14), as a function of the project value, V, in the presence of information costs, λ. r is the interest rate, δ is the opportunity cost of delaying project or a constant payout rate, I denotes the cost of investment or investment expenditure, σ stands for the volatility, λ_C (respectively λ_V) represents the information cost related to G(V) (respectively V). It is assumed that $r = 5.5$ %, $\delta = 6$ %, $I = 500$, and $\sigma = 45$ %. All things being equal, a larger project value can be associated with a greater value of the option to invest. In the presence of the shadow costs of incomplete information regarding project value, the option to invest value increases. In the case where information costs concern the option value, option to expand value drops instead of increasing. It is of interest to note that the negative effect due to incomplete option value information and the positive effect due to incomplete project value information are compensated. But, on the whole, the presence of two types of information costs increases the option to expand value compared to its level in the complete information case.

3. The Value of the Option to Expand

The management can expand the project if economic or technical conditions are favourable. An option to expand is a call option to acquire an additional part to the initial project, where the cost to expand is the exercise price. This managerial flexibility has a value and the cost of expanding could be reduced if flexibility is built into the project at an early stage. The value of this option in the presence of shadow costs of incomplete information can be computed using the following formula:

$$G(V, I, r, \delta, T, \sigma, \lambda_V, \lambda_C) = V\,e^{-(\delta+\lambda_C-\lambda_V)T}N(d_1) - I\,e^{-(r+\lambda_C)T}N(d_2) \qquad (15)$$

with:

$$d_1 = \frac{\left[\ln\left(\frac{V}{I}\right) + \left(r + \lambda_V - \delta + \frac{1}{2}\sigma^2\right)T\right]}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}$$

Table 1
Investment opportunity value G(V)

V	**G(V)**			
	λ_V = 0.00%	λ_V = 1.00%	λ_V = 0.00%	λ_V = 1.00%
	λ_C = 0.00%	λ_C = 0.00%	λ_C = 1.00%	λ_C = 1.00%
300	100,56	115,37	92,03	104,56
350	125,35	142,11	115,65	129,88
400	151,71	170,23	140,97	156,72
450	179,53	199,62	167,86	184,97
500	208,71	230,19	196,24	214,52
550	239,17	261,86	226,02	245,30
600	270,85	294,56	257,14	277,25
650	303,68	328,23	289,54	310,30
700	337,61	362,83	323,16	344,40

Table 2
Time to expand option values

V	**Call Values**			
	λ_V = 0.00%	λ_V = 1.00%	λ_V = 0.00%	λ_V = 1.00%
	λ_C = 0.00%	λ_C = 0.00%	λ_C = 1.00%	λ_C = 1.00%
300	63.86224	76.42005	56.64072	67.77850
350	80.37755	95.78343	71.28849	84.95228
400	97.67687	115.99436	86.63161	102.87777
450	115.62982	136.90720	102.55445	121.42580
500	134,13617	158,41084	118,96811	140,49781
550	153,11718	180,41857	135,80275	160,01692
600	172,50962	202,86120	153,00231	179,92175
650	192,26205	225,68269	170,52115	200,16259
700	212,33196	248,83699	188,32156	220,69861

Table 2 simulates the value of the time to expand option with and without information costs for the option and its underlying asset. It is assumed that I = 500, δ = 6%, r = 5.5%, σ = 45%, and T = 12. This Table shows that the high project values generate an increase in the value of the option to expand. In the presence of the shadow costs of incomplete information regarding project value, the option value increases. In the case where information costs concern the option value, option to expand value decreases. Finally, when we take into account the information costs on both the underlying project and the option, the option to expand value increases.

4. The Value of the Option to Contract

The option to contract has a positive value if market conditions turn weaker than originally expected. In this case, management can then reduce the scale of operations and thus saving part of the planned investment outlays. This analogous to a put option on part of the initial project, with exercise price equal to the potential cost savings. Following Trigeorgis, this option may be particularly valuable in the case of new-product introductions in uncertain markets. The value of the option to contract can be simulated using the following formula.

$$G(V,I,r,\delta,T,\sigma,\lambda_V,\lambda_C) = I\,e^{-(r+\lambda_C)T}N(-d_2) - V\,e^{-(\delta+\lambda_C-\lambda_V)T}N(-d_1) \qquad (16)$$

with:

$$d_1 = \frac{\left[\ln\left(\frac{V}{I}\right)+\left(r+\lambda_V-\delta+\frac{1}{2}\sigma^2\right)T\right]}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}$$

Table 3
Option to contract values

V	Option Values			
	λ_V = 0.00%	λ_V = 1.00%	λ_V = 0.00%	λ_V = 1.00%
	λ_C = 0.00%	λ_C = 0.00%	λ_C = 1.00%	λ_C = 1.00%
1	51.66080	51.65769	45.81902	45.81626
50	50.49407	50.34767	44.78422	44.65438
100	49.38165	49.11073	43.79760	43.55731
150	48.34655	47.96912	42.87954	42.54479
200	47.37949	46.90969	42.02183	41.60516
250	46.47192	45.92108	41.21689	40.72834
300	45.61670	44.99408	40.45838	39.90617
350	44.80790	44.12124	39.74104	39.13203
400	44.04060	43.29644	39.06051	38.40050
450	43.31066	42.51459	38.41311	37.70706
500	42.61457	41.77144	37.79573	37.04794
550	41.94931	41.06335	37.20570	36.41993

Table 3 indicates the value of the option to contract in the presence of information costs for the option and its underlying asset. We investigate the opportunity to contract the scale of the project by 5%,[14] saving an amount of 100. It is assumed that $\delta = 6\%$, r = 5.5%, σ = 45%, and T = 12. The exercise price is the same for all of table 3 and is equal to 100. In this context, the firm has the right, but not the obligation, to undertake the put option at an exercise price corresponding to 100. Table 3 also describes the option to contract values in the case of information uncertainty. As expected, the high project values (i.e. favourable market conditions) generate a decrease in the value of the option to contract: the option to contract has a positive value if market conditions are unfavourable.

5. The Value of the Option to Abandon

Management can abandon current project and resale value of capital equipment. If prices suffer a sustainable decline or the operation does poorly for some other reason, management may have a valuable option to abandon the project in exchange for its salvage value. The option to abandon a project provides partial insurance against failure. The option to abandon can be valued as an American put option on the project's current value, with an exercise price corresponding to the salvage or best alternative use value. Table 4 simulates the values of the option to abandon for different levels of information costs regarding the option and its underlying asset. I is the value received on abandonment and T is the number of years until abandon (years). We use the equation (16) and assume that I = 150, δ = 5%, r = 5%, σ = 40%, and T = 10. In the same spirit as for the option to contract, this Table shows that the value of the option to abandon the project increases when market conditions decline severely (that is, when the value of the project is weak).

Table 4
Option to abandon values

V	**Put Values**			
	$\lambda_V = 0.00\%$	$\lambda_V = 1.00\%$	$\lambda_V = 0.00\%$	$\lambda_V = 1.00\%$
	$\lambda_C = 0.00\%$	$\lambda_C = 0.00\%$	$\lambda_C = 1.00\%$	$\lambda_C = 1.00\%$
1	90.37313	90.30938	81.77299	81.71530
50	66.92000	65.07766	60.55172	58.88470
100	52.58641	50.27457	47.58215	45.49031
150	43.02524	40.62699	38.93085	36.76082
200	36.15249	33.80373	32.71213	30.58688
250	30.96604	28.71878	28.01923	25.98583
300	26.91483	24.78730	24.35355	22.42848
350	23.66723	21.66288	21.41499	19.60139
400	21.01038	19.12596	19.01098	17.30589
450	18.80079	17.03008	17.01166	15.40946
500	16.93798	15.27362	15.32612	13.82014
550	15.34938	13.78377	13.88869	12.47207

6. The Value of the Option to Switch and the Growth Option

In general, investment is a link of interrelated projects opening future growth opportunities. The growth option provides the company with a possibility to make a follow-on investment in the future. It is analogous to a call option. The option to grow is used when an initial investment is required for further development. The project can be considered as a link in a chain of related projects and may serve as a springboard for future project generations. But unless the firm makes that initial investment, subsequent generations will not be feasible. Kester (1984) recognised the importance of the real growth option on firms and argued that the growth option constituted can account for more than half of the market value for most of the companies. The value of the growth option can be computed using formula (15). Table 5 simulates the values of the growth option as a function of project value and information costs. It is assumed that $I = 30$, $\delta = 5\%$, $r = 7\%$, and $\sigma = 35\%$.

Table 5 simulates the values of the growth option for different levels of information costs. The high project values generate an increase in the value of the growth option. In the presence of the shadow costs of incomplete information regarding project value, the option value increases. In the case where information costs concern the option value, the growth option value drops instead of increasing. And the presence of two types of information costs increases the growth option value compared to its level in the complete information case.

Table 5
Growth option prices

V	**Call Values**			
	$\lambda_V = 0.00\%$	$\lambda_V = 1.00\%$	$\lambda_V = 0.00\%$	$\lambda_V = 1.00\%$
	$\lambda_C = 0.00\%$	$\lambda_C = 0.00\%$	$\lambda_C = 1.00\%$	$\lambda_C = 1.00\%$
1	0.00950	0.01411	0.00818	0.01215
25	6.27361	7.79132	5.39975	6.70605
50	16.31713	19.78988	14.04428	17.03331
75	27.22501	32.66339	23.43279	28.11364
100	38.48884	45.88367	33.12765	39.49244
125	49.93540	59.27752	42.97980	51.02064
150	61.48820	72.77038	52.92338	62.63405
175	73.10801	86.32475	62.92464	74.30040
200	84.77264	99.91972	72.96449	86.00170
225	96.46860	113.54277	83.03129	97.72716
250	108.18723	127.18593	93.11761	109.46994
275	119.92274	140.84393	103.21846	121.22550

The managerial flexibility to be able to shut-down and restart operations can be valuable if prices are such that cash revenues are not sufficient to cover variable operating costs. It might be better not to operate temporarily. If prices rise sufficiently, operations can be restarted. Thus, operations in each year may be seen as a call option to acquire that year's cash revenues by paying the variable costs of operating as a strike price. It is equivalent to the firm having a portfolio of call and put options. For example, being able to temporarily shut down a project is equivalent to a put option and restarting operations when the project has been down and become up is equivalent to a call option.

B. The Valuation Procedure in a Discrete-time Setting in the Presence of Information Costs: The Value of the Time-to-build Option

Few investments in practice are a single up-front outlay. However, most investments are sequential and staged into several investments. This creates valuable options to default at any given stage. The completion of one stage gives the right, but not the obligation, to undertake the next stage and the options that this stage provides. The staged investment can be viewed as a series of compound options. In this case, the valuation process can be computed discretely. The project is a perpetual cash flow with a fixed capital outlay. There are points when the project has a positive *NPV*, but we are better off not taking it because the option to undertake the project in the future is more valuable. Since the investment is irreversible, when we take the project, we destroy the value of waiting. It is possible in this context to extend the standard binomial model to account for the effects of information costs. When generating the binomial tree for the underlying asset, we must account for the information cost of the investment opportunity. When we work backwards, we must account for the information cost regarding the option.

In most investments opportunities, management holds an option to defer the life time of investment and see if the cash outflow meets the product price. Some projects could increase in value when new information is available and uncertainty decreased with more favourable conditions.[15] The value of waiting to invest or the option to defer can be seen as an American call option on the gross present value of the future expected cash flows (Trigeorgis (1990, 1993). Using a risk-neutral approach, we adapt the binomial model to account for the effects of incomplete information. The valuation procedure can be described in the following steps. Assuming that the state variable of the project value is the price P of the output, and the project generates a unit per year, the gross present value is:

$$V = \frac{P}{r} \tag{17}$$

The up multiplier, u, and down multiplier, d, are calculated by using formulas in Cox, Ross and Rubinstein (1979):

$$u = e^{\sigma\sqrt{\frac{T}{N}}}, \quad d = e^{-\sigma\sqrt{\frac{T}{N}}} \tag{18}$$

where T is the number of years to expiration and N is the number of binomial periods. These multipliers are used to calculate the future gross value V in the nodes of the binomial tree. The risk neutral probability for the up and down branches is calculated as:

$$p = \frac{e^{(r+\lambda_V-\delta)\frac{T}{N}} - d}{u-d} \tag{19}$$

The discount factor at each node is:

$$e^{-(r+\lambda_C)\frac{T}{N}} \tag{20}$$

The binomial tree should be constructed in such way that it can incorporate the investments needed. We count backward from the end and calculate, in every node, the value by using the binomial formula for one period and subtracting the value of the investment. To consider this in the binomial tree, the value at each node should be the maximum of the value of the project in this node and zero. The calculation of the value in each node should continue in this backward calculation until the value of the firm finally reaches the present time.

Figure 1 simulates the values of the time to build option in the complete information case. It is assumed that the state variable of the project value P = 300, the present value of the cash-flows from the project V = 3000, initial investment I = 800, number of years to maturity T = 6, volatility σ = 40%, risk-free interest rate r = 10%, cash-flow rate generated by the project δ =10%, number of binomial periods N = 6, up multiplier u = 1.49182, down multiplier d = 0.67032, up probability p = 0.40131, down probability 1 – p = 0.59869, discount factor 0.90484, and information costs $\lambda_V = \lambda_C =$ 0%. Columns of the binomial tree have four elements in each node: state variable, project value, NPV (if project is undertaken), and option value.

Figure 2 simulates the values of the time to build option in the presence of information costs for the option and its underlying asset: $\lambda_V = \lambda_C =$ 1%. It is assumed that the state variable of the project value P = 300, present value of the cash-flows from the project V = 3000, initial investment I = 800, number of years to maturity T = 6, volatility σ = 40%, risk-free interest rate r = 10%, cash-flow rate generated by the project δ =10%, number of binomial periods N = 6, up multiplier u = 1.49182, down multiplier d = 0.67032, up probability p = 0.41355, down probability 1 – p = 0.58645, and discount factor 0.89583.

Coherently with our findings in the continuous time setting, time to build option value increases as information costs rise. When information uncertainty concerns the underlying project, λ_V = 1%, option value increases (1339.34). In the case where information costs are related to the option value, λ_C = 1%, the growth option value drops instead of increasing (1168.23). The presence of two types of information costs increases the time to build option value compared to its level in the complete information case (1261.34 for $\lambda_V = \lambda_C$ = 1%, compared to 1240.47 when $\lambda_V = \lambda_C =$ 0%). Our results show that when information uncertainty of expected cash flows rises it increases the value of flexibility. This gives support to the well-known relation between the general uncertainty and the value of flexibility.

III. CONCLUSION

This paper reviews the main concepts in real options and extends the literature for the valuation of real options in the presence of information costs. These options are fundamental in the valuation process of investments and capital budgeting. However, they are valued in a standard framework ignoring the role of information costs in investment decisions. Information costs play a central role in the capital budgeting process, since managers do not invest in projects they do not know about. When money is engaged in research and development, in project analysis and valuation, it is natural to require a return that accounts for these expenses. Therefore, information costs or shadow costs of incomplete information represent an appropriate component of the discount rate in investment decisions.

Figure 1:
Time to build binomial tree using binomial approach

						3306.95291 33069.52914 32269.52914 32269.52914
					2216.71683 22167.16830 21367.16830 19333.81339	
				1485.90973 14859.09727 14059.09727 11510.61530		1485.90973 14859.09727 14059.09727 14059.09727
			996.03508 9960.35077 9160.35077 6786.15476		996.03508 9960.35077 9160.35077 8288.62814	
		667.66228 6676.62279 5876.62279 3939.21806		667.66228 6676.62279 5876.62279 4811.37180		667.66228 6676.62279 5876.62279 5876.62279
	447.54741 4475.47409 3675.47409 2238.35236		447.54741 4475.47409 3675.47409 2722.85818		447.54741 4475.47409 3675.47409 3325.70649	
300.00000 3000.00000 2200.00000 1240.47432		300.00000 3000.00000 2200.00000 1491.43732		300.00000 3000.00000 2200.00000 1801.20766		300.00000 3000.00000 2200.00000 2200.00000
	201.09601 2010.96014 1210.96014 789.48966		201.09601 2010.96014 1210.96014 927.99068		201.09601 2010.96014 1210.96014 1095.72204	
		134.79869 1347.98689 547.98689 457.64901		134.79869 1347.98689 547.98689 505.67512		134.79869 1347.98689 547.98689 547.98689
			90.35826 903.58264 103.58264 222.76435		90.35826 903.58264 103.58264 198.98633	
				60.56896 605.68955 -194.31045 72.25640		60.56896 605.68955 -194.31045 0.00000
					40.60058 406.00585 -393.99415 0.00000	
						27.21539 272.15386 -527.84614 0.00000

Figure 2:
Time to build binomial price and standard NPV

						3306.95291 33069.52914 32269.52914 32269.52914
					2216.71683 22167.16830 21367.16830 19341.01602	
				1485.90973 14859.09727 14059.09727 11523.58486		1485.90973 14859.09727 14059.09727 14059.09727
			996.03508 9960.35077 9160.35077 6803.67035		996.03508 9960.35077 9160.35077 8295.83076	
		667.66228 6676.62279 5876.62279 3960.24496		667.66228 6676.62279 5876.62279 4824.34136		667.66228 6676.62279 5876.62279 5876.62279
	447.54741 4475.47409 3675.47409 2260.72925		447.54741 4475.47409 3675.47409 2740.37377		447.54741 4475.47409 3675.47409 3332.90911	
300.00000 3000.00000 2200.00000 1261.33910		300.00000 3000.00000 2200.00000 1510.53373		300.00000 3000.00000 2200.00000 1814.17722		300.00000 3000.00000 2200.00000 2200.00000
	201.09601 2010.96014 1210.96014 806.69474		201.09601 2010.96014 1210.96014 942.79299		201.09601 2010.96014 1210.96014 1102.92467	
		134.79869 1347.98689 547.98689 470.31867		134.79869 1347.98689 547.98689 515.25467		134.79869 1347.98689 547.98689 547.98689
			90.35826 903.58264 103.58264 230.39831		90.35826 903.58264 103.58264 203.01204	
				60.56896 605.68955 -194.31045 75.20963		60.56896 605.68955 -194.31045 0.00000
					40.60058 406.00585 -393.99415 0.00000	
						27.21539 272.15386 -527.84614 0.00000

We introduce information costs in the spirit of Merton (1987) and Bellalah (1999, 2001) in the capital budgeting process and real options valuation. We suggest a general derivation for the valuation of real options when the underlying asset is observable and when it is not observable. This provides a generalisation of the Black-Scholes (1973) formula, the Merton (1998) formula and the binomial approach which accounts for the effects of incomplete information. We examine the valuation of the option to invest, the option to expand, the option to contract, the option to abandon, the switch option and the growth option, the option to shut down and restart, the option to defer, and the time to build option in the presence of information costs. Simulation results are provided using reasonable values for information costs. Our analysis can be extended to other types of real options. In particular, it can be applied to compound real options and "exotic" real options. The analysis can also be tested using real data. It is also possible to extend our study to account for stochastic volatility of cash flows.

ENDNOTES

1. For a survey of these techniques, the reader can refer to Smith and Nau (1994), Lee (1988), Agmon (1991) among others.
2. Merton's model may be stated as follows: $R_V - r = \beta_V [R_m - r] + \lambda_V - \beta_V \lambda_m$ where R_V: the equilibrium expected return on an asset V, R_m: the equilibrium expected return on the market portfolio, r : one plus the riskless rate of interest, $\beta_V = cov(R_V/R_m)/var(R_m)$, λ_V: the equilibrium aggregate "shadow cost" for the asset V. It is of the same dimension as the expected rate of return on this asset V, λ_m: the weighted average shadow cost of incomplete information over all assets.
3. For a survey of the literature on real options, the reader can refer to Trigeorgis (1990, 1993), Pindyck (1991), Padock, Siegel and Smith (1988), Newton (1996), Myers (1984), Myers and Majd (1990) among others.
4. We concentrate our analysis in the option to invest, the option to expand, the option to contract, the option to abandon, the option to switch and the growth option, the option to shut down and restart operations, the option to defer, and the time to build option. These real options are studied in different contexts by Kogut (1991), Kogut and Kulatilaka (1994a, b), Mac Donald and Siegel (1984, 1986), Brennan and Schwartz (1985), Berger, Ofek and Swary (1996) among others. Several other real options exist, but the same analysis applies.
5. These options appear in the work of Dentskevich and Salkin (1991), Dixit (1992, 1995), Dixit and Pindyck (1994, 1995), Faulkner (1996) and Ingersoll and Ross (1992) among others.
6. Myers (1977) shows that the value of a firm is the combined value of the assets already in use and the present value of the future investment opportunities.
7. A typical example is firms in the oil and gas exploration and production business. Other examples include power stations and pharmaceutical companies. See for example Paddock, Siegel and Smith (1988).
8. Two types of flexibility are present in the project: internal and external flexibility. The internal flexibility corresponds to the manager's flexibility to modify the project. This can include expansion, alteration, abandonment, etc. The external flexibility corresponds to the growth option which gives the possibility to perform another project.
9. For a survey of the literature on standard options and exotic options pricing, the reader can refer to Cox, Ross and Rubinstein (1979), Cox and Ross (1976), Black and Scholes (1973), among others.
10. Or the lost value in time.
11. Dixit & Pindyck (1994, 1995) argue that the risk free interest rate is useful for three types of real economic problems. Dixit, Pindyck and Sodal (1999) use an exogenous discount rate for incomplete markets analysis. Firstly, in complete markets, by changing the probability measure, any stochastic process can be transformed to a risk-neutral one. Secondly, economic applications assume that firms are risk-neutral even when investors and stockholders are risk-averse.

Thirdly, there is no correlation between the market portfolio and macroeconomic shocks.

12. Dixit and Pindyck (1994) explain that the time to maturity is defined by the expiration of the patent. After the expiration, the firm loses the opportunity to gain a competitive advantage due to the patent.
13. By using the model of Merton (1987), Bellalah and Jacquillat (1995) and Bellalah (2001) deduce the financial option value in the context of incomplete information.
14. We assume here that the firm has the possibility to abandon 5% of the investment value.
15. Ingersoll and Ross (1992) showed that the option to defer is reversible and more valuable when there is high economic uncertainty and long investment horizons.

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